

## The Frobenius group determinant, factored for the permutation group $S_3$

$$\text{In[2]:= } \mathbf{Det} \begin{bmatrix} \mathbf{x_1} & \mathbf{x_{12}} & \mathbf{x_{23}} & \mathbf{x_{13}} & \mathbf{x_{123}} & \mathbf{x_{132}} \\ \mathbf{x_{12}} & \mathbf{x_1} & \mathbf{x_{132}} & \mathbf{x_{123}} & \mathbf{x_{13}} & \mathbf{x_{23}} \\ \mathbf{x_{23}} & \mathbf{x_{123}} & \mathbf{x_1} & \mathbf{x_{132}} & \mathbf{x_{12}} & \mathbf{x_{13}} \\ \mathbf{x_{13}} & \mathbf{x_{132}} & \mathbf{x_{123}} & \mathbf{x_1} & \mathbf{x_{23}} & \mathbf{x_{12}} \\ \mathbf{x_{123}} & \mathbf{x_{23}} & \mathbf{x_{13}} & \mathbf{x_{12}} & \mathbf{x_{132}} & \mathbf{x_1} \\ \mathbf{x_{132}} & \mathbf{x_{13}} & \mathbf{x_{12}} & \mathbf{x_{23}} & \mathbf{x_1} & \mathbf{x_{123}} \end{bmatrix} // \mathbf{Factor}$$

$$\text{Out[2]= } - (x_1 - x_{12} - x_{13} - x_{23} + x_{123} + x_{132}) (x_1 + x_{12} + x_{13} + x_{23} + x_{123} + x_{132}) (x_1^2 - x_{12}^2 + x_{12} x_{13} - x_{13}^2 + x_{12} x_{23} + x_{13} x_{23} - x_{23}^2 - x_1 x_{123} + x_{123}^2 - x_1 x_{132} - x_{123} x_{132} + x_{132}^2)^2$$

Can you spot the three representations, trivial, alternating and 2-dimensional?

Is it a coincidence that  $6 = 1^2 + 1^2 + 2^2$ ?