Hints for the additional exercises

Exercise 2. For one hole see Exercise 4.8c from Thorpe. More generally if you have two surfaces given by f = 0 and g = 0 then what is the surface given by the equation f g = 0?

Exercise 9. In this exercise you never have to actually solve a differential equation. Just verifying your guess is actually a solution should suffice. All the guessing comes from the intuition that the map W_α actually preserves distances so that it transfers the easy geometry of the plane directly onto the cone.
For part c. it may be helpful to collect the cosine and sine into a complex exponential, writing

$$W_\alpha(\gamma(t)) = \frac{r(t)}{\sqrt{1+a^2}}(1, ae^{i\theta(t)\frac{\sqrt{1+a^2}}{a}})$$

where r and θ are the absolute value and argument of your parameterized straight line $\gamma(t)$. Notice I'm identifying $\mathbb{R}^3$ with $\mathbb{R} \times \mathbb{C}$. Maybe you can even make U_α sit in the complex plane, then the argument is just the imaginary part of the logarithm.