

Geometry Tutorial 5

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1. Imagine sets A, B and a function $f : A \rightarrow B$.
 - (a) Prove that $a \sim a'$ iff $f(a) = f(a')$ defines an equivalence relation
 - (b) C is the set of equivalence classes of A with respect to the equivalence relation above. Explain why there is a well-defined map $g : C \rightarrow B$ defined by $g(c) = f(a)$ if $a \in c \in C$.
 - (c) Assuming f is surjective prove that g is a bijection.
2. Derive the relation $\angle(a, b) + \angle(b, c) = \angle(a, c)$ from the general definition of angle addition given in the lecture notes: $\angle(u, v) + \angle(z, w) = \Phi^{-1}(\Phi(\angle(u, v)) \circ \Phi(\angle(z, w)))$
3. For four points A, B, C, D in the plane is it true that

$$\angle ADB + \angle BAC + \angle CBD + \angle DCA$$

is the zero angle?

4. If $u, v \in E$ are unit vectors spanning linear subspaces U, V we consider the composition of reflections $r = s_V \circ s_U$.
 - (a) Prove that r is a rotation.
 - (b) Recall the angle of a rotation r is the unique oriented angle $\angle(a, b)$ such that $\Phi\angle(a, b) = r$. Prove that this angle is twice $\angle(u, v)$.