Computing the colored Jones polynomial by using solvable Lie algebras

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The goal of this spring school is to study the recent seminal paper of Dror Bar-Natan and Roland van der Veen « A polynomial time knot polynomial ».
The new polynomials can be seen as closer relatives of the famous Alexander polynomial than other quantum invariants as e .g. the Jones polynomial, because they can be calculated along the same lines as the Alexander polynomial and in merely poly-time complexity. Therefore we start with a recapitulation of the various definitions of the Alexander polynomial, which could perhaps be generalized one day for the new polynomials too. We continue with a broad introduction into the construction of quantum knot invariants, in particular of the colored Jones polynomial. We switch then to the very promising alternative approach to quantum representations, namely the Lawrence-Krammer-Bigelow homological representations of braid groups. Finally, we come to the new knot polynomials defined by Bar-Natan and van der Veen.

Here is the list of talks, together with a time proposal.

A) (2h) Different ways to define the Alexander-Conway polynomial of links in 3-space without using representation theory: Jean-Mark Hok + Virginie Bonnier

1. topological using the Alexander ideal , [4]
2. as a Reidemeister torsion, [12]
3. by a skein relation, [7]
4. as a state sum, [7]
5. using Fox calculus, [5]
6. using the Seifert form, [4]

B) (2h) Different ways to define the Alexander-Conway polynomial of links in 3-space by using representation theory: Fathi Ben Aribi + Giulio Belletti + Konstantinos Karvounis

1. with the Burau representation, [7]
2. with the Lawrence-Krammer-Bigelow representation, [6] and [9]
3. as a canonical Vassiliev power series, [1] and [2]
4. using the Weil algebra, [3]

C) (1,5h) Young-Baxter equations and braid groupe representations, Hopf algebras and monoidal categories, [8] Sonny Willets

D) (1,5h) Drinfeld quantum double and the quantized enveloping algebra U_q sl(2), [8] , Sjabbo Schaveling

E) (1,5h) Representation theory of U_q sl(2), quantization of an infinitesimal symmetric category and Drinfeld associators, [8], Ajinkya Kulkarni

F) (1,5h) The colored Jones function, Vassiliev invariants and the Melvin-Morton-Rozansky expansion of the colored Jones polynomial, [10] and [11], Martin Palmer-Anghel

G) (3h) The R-matrix expansion and the calculation of the perturbative expansion of the colored Jones polynomial, [11], Roland van der Veen

H) (3h) Generic quantum sl_2 representations are isomorphic to the Lawrence-Krammer-Bigelow representations, [9], Jules Martel

I) (1,5h) A topological formula of the loop expansion of the colored Jones polynomial by using the Lawrence-Krammer-Bigelow representations and a short proof of the Melvin-Morton-Rozansky conjecture, [6], Cristina Palmer-Anghel

J) (6h) A new polynomial time knot polynomial, [3], Dror Bar-Natan

References :

1. Bar-Natan : On the Vassiliev knot invariants
2. Bar-Natan, Garoufalidis : On the Melvin-Morton-Rozansky conjecture
3. Bar-Natan, van der Veen : A polynomial time knot polynomial
4. Burde, Zieschang : Knots
5. Crowell, Fox : Introduction to knot theory
6. Ito : Topological formula of the loop expansion of the colored Jones polynomial
7. Kauffman : Formal knot theory
8. Kassel, Rosso, Turaev : Quantum groups and knot invariants
9. Kohno : Quantum and homological representations of braid groups
10. Melvin, Morton : The colored Jones function
11. Overbay : Perturbative expansion of the colored Jones polynomial
12. Turaev : Introduction to combinatorial torsions