Homework exercises Algebraic topology, hand in before class on 10-10-2018

Exercise 1. (Double Möbius)
Recall that the Möbius strip M is the quotient space $[-1,1]^2$ where the pairs of points $(1,y),(-1,-y)$ are identified. Define F to be the result of identifying two copies M_1, M_2 of M along their boundaries. More precisely if (x_i, y_i) denotes a point in M_i then we start with the disjoint union $M_1 \sqcup M_2$ and identify $(x_1, 1)$ with $(x_2, 1)$ and $(x_1, -1)$ with $(x_2, -1)$.

a. Show that M and S^1 are homotopy equivalent by giving explicit homotopies.

b. Calculate the homology groups of M.

c. Use the Mayer-Vietoris sequence (Exercise sheet 5 exercise 1) to compute the homology groups of F.

d. (Bonus) What surface is F?

Exercise 2. (Weighing in on the barycenter)

a. Start with an n-simplex and iterate the barycentric subdivision n times. How many simplices do you get?

b. Recall that we used the notation $[v_0, v_1, \dots, v_n]$ for the convex hull of the vectors $v_0, \dots v_n$ also known as an affine n-simplex. Prove that the simplices in the barycentric subdivision of $[v_0, v_1, \dots, v_n]$ are in bijection with chains $F_0 \subset F_1 \subset F_2 \subset \dots \subset F_n = [v_0, \dots v_n]$ where $F_i = [v_{i_0}, v_{i_1}, \dots, v_{i_i}]$.

c. Is there an affine 2-simplex such that is similar to a 2-simplex in its own barycentric subdivision? Two subsets in $\mathbb{R}^n$ are said to be similar if they are related by a composition of translations, multiples of the identity and elements of $O(N)$. For triangles this just means equal angles.