Homework exercises Algebraic topology, hand in before class on 3-10-2018

Exercise 1. (Prism operator)

a. For the maps $f, g : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (x, y)$ and $g(x, y) = (x + 1, y)$, give a linear map G that is a homotopy between f and g.

b. Define the singular 2-simplex $\sigma : \Delta^2 \to \mathbb{R}^2$ by $\sigma(e_0) = 0$, $\sigma(e_1) = e_1$, $\sigma(e_2) = e_2$. Describe $f_*\sigma$ and $g_*\sigma$ explicitly and draw their images.

c. Referring to the lecture of 26-9 (final hour) describe explicitly and draw the tetrahedra corresponding to the images of the summands of $P_2(\sigma) = \sum_i (-1)^i G_*(\sigma \circ \sigma_i, \alpha_i)$.

d. Explain how your pictures from part b. fit in the pictures from part c.

e. If homotopic maps are supposed to give identical results on homology, how come $f_*\sigma$ and $g_*\sigma$ differ by more than just a boundary?

Exercise 2. $(18 \neq 19)$
Prove that Δ^{18} is not homeomorphic to Δ^{19}. You may make use of the excision theorem and its consequences as treated in the lectures.