Homework exercises Algebraic topology, hand in before class on 19-9-2018

Exercise 1.
Compute $H_0(X;\mathbb{Z}/2\mathbb{Z})$ where X is the finite set $\{a,b\}$ whose topology consists of the following set subsets $\{\emptyset,\{a\},X\}$.

Exercise 2.

1. For any $p,q\in\mathbb{Z}$ give an element $s_{p,q}\in C_2(\mathbb{R}^2;\mathbb{Z})$ such that $s_{p,q}$ is non-zero on only two singular 2-simplices α,β and $im(\alpha)\cup im(\beta)=[p,p+1]\times[q,q+1]$ and $\partial_2(s_{p,q})\in\mathbb{Z}[S(\mathbb{R}^2)_1]$ is the function that sends all singular 1-simplices in $\mathbb{R}^2$ to 0 except the simplices $a,b,c,d:\Delta^1\to\mathbb{R}^2$, defined below. $\partial_2(s_{p,q})$ should send a,b,c to 1 and d to -1. Define

$$\begin{aligned}
a(te_0+(1-t)e_1)&=pe_0+qe_1+te_0\\
b(te_0+(1-t)e_1)&=pe_0+qe_1+e_0+te_1\\
c(te_0+(1-t)e_1)&=pe_0+qe_1+e_0+e_1-te_0\\
d(te_0+(1-t)e_1)&=pe_0+qe_1+te_1
\end{aligned}$$

2. Identifying elements of $\mathbb{Z}[S(\mathbb{R}^2)_n]$ with formal $\mathbb{Z}$-linear combination of n-simplices, how would you write $s_{0,0}$ and its boundary $\partial_2 s_{0,0}$?

3. Finally express the homology class of $\partial_2\sum_{p,q=0}^{7}s_{p,q}$ as the class of a function that takes only four non-zero values.
Hint. Draw a picture.